A quiet-time empirical model of equatorial plasma drift in the Peruvian sector

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Abstract. Equatorial vertical plasma drift is an important consequence of the E and F region dynamos. Understanding the climatology of vertical drift can provide significant insight into ionospheric phenomena, such as the equatorial ionospheric anomaly. In this study we present the first empirical model of vertical plasma drifts observed by the JULIA (Jicamarca Unattended Long-term studies of the Ionosphere and Atmosphere) coherent scatter radar located in Peru. The model, called JVDM (JULIA Vertical Drift Model), describes the local time, seasonal and solar flux behavior of the equatorial vertical drifts in the Peruvian sector. The model is valid from 0800 through 1600 local time, which is typically when JULIA makes vertical drift measurements. During very high solar flux conditions however, the model is unreliable before 1000 local time, due to a lack of JULIA data. The model includes a climatology of the equatorial vertical drifts, as well as an estimate of the day-to-day variability, which can be significant. The day-time drifts typically peak between 1000 and 1200 LT and have amplitudes of 25-30 m/s $\pm$ 10 m/s. The model has been validated against the global empirical model of Scherliess and Fejer, with a total rms difference of under 4 m/s for 1000 to 1600 LT. This model will allow researchers to study daily variations in the equatorial electric field by subtracting the climatological mean. Model coefficients and software are available online at http://geomag.org/models and http://www.earthref.org.

1. Introduction

Equatorial vertical plasma drifts are driven by complex dynamo processes in the ionospheric E and F regions. Neutral winds on the day-side cause positive and negative charges to accumulate at the dawn and dusk terminators respectively, giving rise to the equatorial electric field (EEF) [*Heelis*, 2004]. This field is eastward on the day-side and westward on the night-side. The EEF in combination with the earth's magnetic field drives ion drifts which are typically upward and westward during the day-time and downward and eastward at night.

Accurately measuring and predicting vertical plasma drifts is important for the study of many physical processes in the low latitude ionosphere, including the Equatorial Ionization Anomaly (EIA) [*Appleton*, 1954], upper F region electron density structures [*Su et al.*, 1995], the equatorial electrojet (EEJ) [*Forbes*, 1981; *Alken and Maus*, 2007] as well as the forecasting of low latitude ionospheric weather. There have been previous studies to model quiet-time vertical plasma drifts using observatory data as well as satellite data. *Scherliess and Fejer* [1999] combined measurements from the Jicamarca incoherent scatter radar (ISR) with observations from the Atmospheric Explorer E (AE-E) satellite to produce a quiet-time empirical vertical drift model at all longitudes, seasons, local time and solar flux values. *Fejer et al.* [2008] produced a quiet-time empirical drift model of observations from the ROCSAT-1 satellite with global longitudinal coverage. There have also been other regional empirical drift models developed [*Abdu et al.*, 1995; *Sastri*, 1996; *Batista et al.*, 1996].

The Jicamarca Radio Observatory (JRO) (11.95°S, 76.87°W) near Lima, Peru has been measuring equatorial vertical plasma drifts at 150 km altitude since 1996 using the JULIA radar. JULIA is a coherent scatter radar which makes high-quality observations of Doppler 150-km echoes which have been shown to yield good estimates of F region vertical ion drifts [*Kudeki and Fawcett*, 1993; *Woodman and Villanueva*, 1995]. JULIA measures 150-km vertical drifts during day-time hours at 5 minute intervals. JULIA data has enabled the study of many important ionospheric processes and is especially useful due to its near-continuous day-time observations of vertical drifts since 1996. However, there has never been an empirical model created for the JULIA data. The model of *Scherliess and Fejer* [1999] used the incoherent scatter radar at Jicamarca which normally operates in campaign mode and does not have continuous measurements at all local times and seasons. In this work we present the first empirical vertical drift model based on the JULIA coherent scatter radar. This model is of interest for studies of electrodynamic effects of atmospheric tides. It also provides the climatology of the equatorial plasma fountain, which is the source of the equatorial ionization anomaly. Last but not least, the model allows users of JULIA data to subtract the climatological mean in order to study daily variations in the eastward electric field.

2. Climatological Mean Model Description

This empirical vertical plasma drift model was derived from JULIA coherent scatter radar data in the period of August 2001 through July 2008. Only quiet-time data (Kp ≤ 3) were used, in the local time sector of 0800 to 1600 when JULIA data are typically available. Figure 1 shows the distribution of data over season, local time, and solar flux level. There is a lack of data in the early morning (0800-1000 LT) during high solar flux at all seasons and also during June and December solstice at lower solar flux levels.

The data selection yields a total of 46,669 drift measurements. Each drift measurement is provided with an error estimate, enabling a weighted least squares fit. The model is a function of local time, season and solar activity. We use the EUVAC flux index [*Richards et al.*, 1994] for the solar activity dependence which is defined as $P = (F10.7 + F10.7A)/2$ where $F10.7A$ is the 81 day average of F10.7. The functional form of the model is given by

$$v(t,s,p) = \sum_{i=1}^{N_t} \sum_{j=1}^{N_s} \sum_{k=1}^{2} a_{ijk} B_i(t) f_j(s) p^{k-1} \tag{1}$$

where t is local time, s is season (day of year), and p is the EUVAC solar flux proxy defined above. The coefficients a_{ijk} are to be determined and the basis functions are given by

$$B_i(t) = \text{ith cubic B-spline with uniform knots from 8 to 16 LT} \tag{2}$$

$$f_j(s) = \begin{cases} \cos\left(\frac{(j-1)\pi s}{365.25}\right) & \text{j odd} \\ \sin\left(\frac{j\pi s}{365.25}\right) & \text{j even} \end{cases} \tag{3}$$

B-splines [*De Boor*, 2001] were chosen for the local time dependence since there does not appear to be a more physically natural basis available. The model of *Scherliess and Fejer* [1999] accounted for seasonal variation by essentially binning their data into broad seasonal bins and performing separate fits for each season. The seasonal structure of the vertical drifts is complex and not fully understood, but does have a periodic structure, most easily observed with peaks during equinox and minima during solstice [*Alken and Maus*, 2007; *Fejer et al.*, 2008]. The oscillatory seasonal basis functions were chosen due to this observed periodicity. A linear fit was performed in the solar flux variable due to the known correlation between the vertical plasma drift and solar activity.

The values of N_t and N_s were chosen by examining how the residual mean square changes as a function of these parameters. The residual mean square is defined as

$$s^2 = \frac{1}{n-p} \sum_{i=1}^{n} w_i \left[v_i - v(t_i, s_i, p_i)\right]^2 \tag{4}$$

where n is the total number of drift measurements and p is the number of parameters in the model ($p = 2N_t N_s$). The weights are $w_i = 1/(\sigma_i^2 + 1)$ with units of $(m/s)^{-2}$ and the values σ_i are the given error estimates with the JULIA data (in m/s). The additive factor of 1 $(\mathrm{m/s})^2$ in the denominator ensures numerically reasonable weights when σ_i is small. The v_i are the JULIA drift observations and $v(t_i, s_i, p_i)$ is the corresponding model value. As more basis functions are added to the model, eventually causing over-fitting to the data, s^2 will approach the true value of the error variance σ^2 [*Draper and Smith*, 1981].

Figure 2 shows the residual mean square s^2 as a function of N_t and N_s individually while holding the other value constant. In both plots the residual mean square decreases as the number of basis functions increases. This is most evident in the N_s plot where s^2 decreases sharply as N_s increases. The N_t plot does not contain such a sharp decrease, however we do see the typical typical asymptotic behavior as over-fitting takes place and s^2 approaches the true variance σ^2. The N_s plot does not approach an asymptotic value of s^2 within a reasonable choice of N_s, which is most likely due to the high degree of variability in the seasonal structure of the data. We therefore choose $N_s = 11$ which should adequately describe the seasonal changes of the JULIA vertical drift data. This choice represents sinusoidal basis functions up to degree 5 in the model, which will satisfactorily capture the peaks during equinox and minima during solstice, as well as allow for differences between March and September equinox and other smaller seasonal structures.

From the N_t figure alone, it is difficult to pinpoint the ideal value of the N_t parameter, and so we computed the Mallows C_p statistic [*Draper and Smith*, 1981] for each of a set of possible N_t values. The Mallows C_p is defined as

$$C_p = RSS_p/\sigma^2 - (n - 2p) \tag{5}$$

where RSS_p is the residual sum of squares for a model with p parameters, σ^2 is the best estimate of the actual error variance, and n is the number of data points. For an accurate model, C_p has an expected value of approximately p. We estimated the error variance as $\sigma^2 = 34.2$ using the data in Figure 2 for $N_s = 11$ and computed the C_p statistic for several possible values of N_t. A plot of C_p vs p is given in Figure 3. Any model with a C_p value close to p passes the C_p test, and we see from the figure that the models with $N_t \geq 7, N_s = 11$ lie on the line $C_p = p$. However the model with $N_t = 7$ was chosen to attempt to keep higher frequency artifacts in the local time dependence of the model to a minimum. This model has 154 parameters and a C_p of 138.3. The discrepancy between these two numbers is largely a result of the accuracy of the estimate of σ^2.

Once suitable values of N_t and N_s were selected for the model in Eq. 1, a weighted least squares regression was performed to minimize the residual sum of squares $RSS = s^2(n-p)$. The calculated value of the coefficient of determination is $R^2 = 0.32$, indicating that the climatological model accounts for 32% of the variation about the mean in the data. This low figure clearly indicates the high degree of day-to-day variability in the equatorial vertical drifts.

Figure 5 shows, as a function of local time and season, the vertical drift values produced by the model as compared with the raw JULIA data and the Scherliess and Fejer model. The JULIA data plot in the middle was created by binning the dataset into half-hour bins in local time and 20 day bins in season, computing the mean of each bin, and then fitting a surface using a continuous curvature gridding algorithm [*Wessel and Smith*, 1991] with a grid spacing of 0.3 hours in local time and 5 days in season. We see a very good agreement between the model and data. All major features of the vertical plasma drifts have been reproduced. The drift maxima during March and September equinox and minima during June and December solstice agree well. We also see that the local time behavior is well reproduced, with maxima near 1030-1130 LT. An important improvement in the JVDM model compared to the Scherliess and Fejer model is the allowance of differences between March and September equinox. The model of Scherliess and Fejer treated both equinoxes as identical but the JULIA data indicates slightly higher drift velocities during September and some asymmetries between the equinoxes at later local times. These seasonal differences are illustrated in Figure 4 which clearly indicates that the seasonal dependence of the Scherliess and Fejer model is insufficient to fully capture the JULIA seasonal drift structure. The JVDM seasonal basis functions allow for more accurate modeling of these features.

We examined the weighted residuals

$$r_i = \sqrt{w_i}(v_i - v(t_i, s_i, p_i)) \tag{6}$$

and do not find any systematic trend which would indicate an insufficient number of terms in the model. There are a few outliers in the residuals which indicate data points that are not typical of the majority of the data. These could be due to errors in those observations or unusual ionospheric conditions when the measurement was made. Since it is difficult to determine the exact cause of these outliers, we choose not to reject them out of hand in the analysis. The weighted mean of all residuals is 1.0×10^{-13}.

3. Model Validation

To validate the JULIA vertical drift model (JVDM), we compared it with the empirical quiet-time vertical drift model of *Scherliess and Fejer* [1999]. This model was based on equatorial vertical drift data from the incoherent scatter radar (ISR) at Jicamarca as well as observations from the Atmospheric Explorer E satellite. The model used Jicamarca ISR data averaged from about 300 to 400 km altitude. Although JULIA makes measurements at 150 km altitude, the comparison is meaningful since it has been found that vertical drift velocity gradients are small. *Pingree and Fejer* [1987] analyzed Jicamarca ISR data and found only small gradients within $|0.05|$ m s^{-1} km^{-1}. *Fejer et al.* [1995] also did not find significant gradients at F region altitudes using the AE-E satellite database.

When computing the rms difference between the two models, defined as

$$\epsilon = \sqrt{\frac{1}{V} \int \left[v(t,s,p) - v_{SF}(t,s,p) \right]^2 dt ds dp} \tag{7}$$

where the integral is taken over local times 0800 to 1600, all seasons and all solar flux conditions, the result is 4.3 m/s. In the above expression, v_{SF} is the vertical drift model of *Scherliess and Fejer* [1999] and v is the JULIA vertical drift model. V is the total volume of parameter space. The primary contribution to this rms difference is most likely the lack of JULIA data before 1000 local time, as well as the different seasonal dependence of the two models. Recomputing the rms difference for the local time sector 1000 to 1600 yields a value of 3.8 m/s. Computing the rms difference for 0800 to 1600 local time, but only up to a maximum EUVAC index of 150 yields a value of 3.4 m/s. As mentioned above, the seasonal dependence of the two models was treated differently which is also contributing error, however these values are fairly reasonable and lead us to conclude this is a positive validation of JVDM with the understanding that there are inaccuracies at high solar flux conditions (> 150) before 10 am local time.

4. Deviation Model

Since the equatorial vertical plasma drifts are highly variable from day-to-day, the climatological mean by itself is not sufficient to describe them. We would therefore like an estimate of the standard deviation from the climatological mean. Since each measurement is taken at a specific local time, season and solar flux and is the only measurement for those particular parameters, the only variability estimate available is the absolute deviation from the mean

$$D_i = |v_i - v(t, s, p)|, \tag{8}$$

which is not the same as the standard deviation. However, if the data is normally distributed, it can be shown that the absolute mean deviation is related to the standard deviation by a constant:

$$D = \sqrt{\frac{2}{\pi}}\sigma. \tag{9}$$

Therefore we present a model of the day-to-day absolute deviation of the vertical drifts from their climatological mean. This model is then used to show that the vertical drifts are in fact normally distributed, so that the standard deviation can be estimated. The same functional form was used as for the mean model:

$$D(t,s,p) = \sum_{i=1}^{N_t}\sum_{j=1}^{N_s}\sum_{k=1}^{2} b_{ijk} B_i(t) f_j(s) p^{k-1} \tag{10}$$

with the same basis functions for each parameter. We performed a similar analysis to that shown in Figure 2 and found the same values of N_t and N_s were suitable as for the mean fit. We then performed an unweighted least squares fit to the model in Eq. 10. In order to show that this model is meaningful, we computed "normalized drifts":

$$\bar{v}_i = \frac{v_i - v(t,s,p)}{\sigma(t,s,p)} = \frac{v_i - v(t,s,p)}{\sqrt{\frac{\pi}{2}} D(t,s,p)} \tag{11}$$

which, if the mean and deviation models are correct, will produce a dataset normally distributed with zero mean and unit standard deviation. Indeed, the mean of the $\bar{v}_i$ dataset is 0.057 and its standard deviation is 1.006. To show that the $\bar{v}_i$ are normally distributed, we computed a quantile-quantile plot in Figure 6a which has normal quantiles on the horizontal axis with the $\bar{v}_i$ quantiles on the vertical axis. Since most of these lie on the line $y = x$ the normalized drift dataset is almost surely normally distributed, since other distributions would result in a deviation from this line. This indicates that our relation between the standard deviation and absolute mean deviation models is correct. This is further illustrated in Figure 6b which shows the probability density profile of the $\bar{v}_i$ dataset along with an ideal normal distribution with zero mean and unit deviation. There is a very good agreement between the two probability functions. Figure 6c shows an example local time profile along with its $\pm$ standard deviation curves to illustrate the high day-to-day variability of the vertical plasma drifts from their climatological mean.

5. Conclusion

We have presented the first empirical model of vertical plasma drifts measured from 150-km JULIA radar echoes. The model includes two components, one for the climatological mean as a function of local time, season and solar activity, and one which provides an estimate of the day-to-day variability of the drifts, as a function of the same parameters. The model has been validated against the global empirical model of *Scherliess and Fejer* [1999] with good agreement. This model also incorporates for the first time the complicated seasonal structure of the drifts, especially the differences between March and September equinox which are not represented in the model of Scherliess and Fejer. Model coefficients and software are available online at http://geomag.org/models and http://www.earthref.org.

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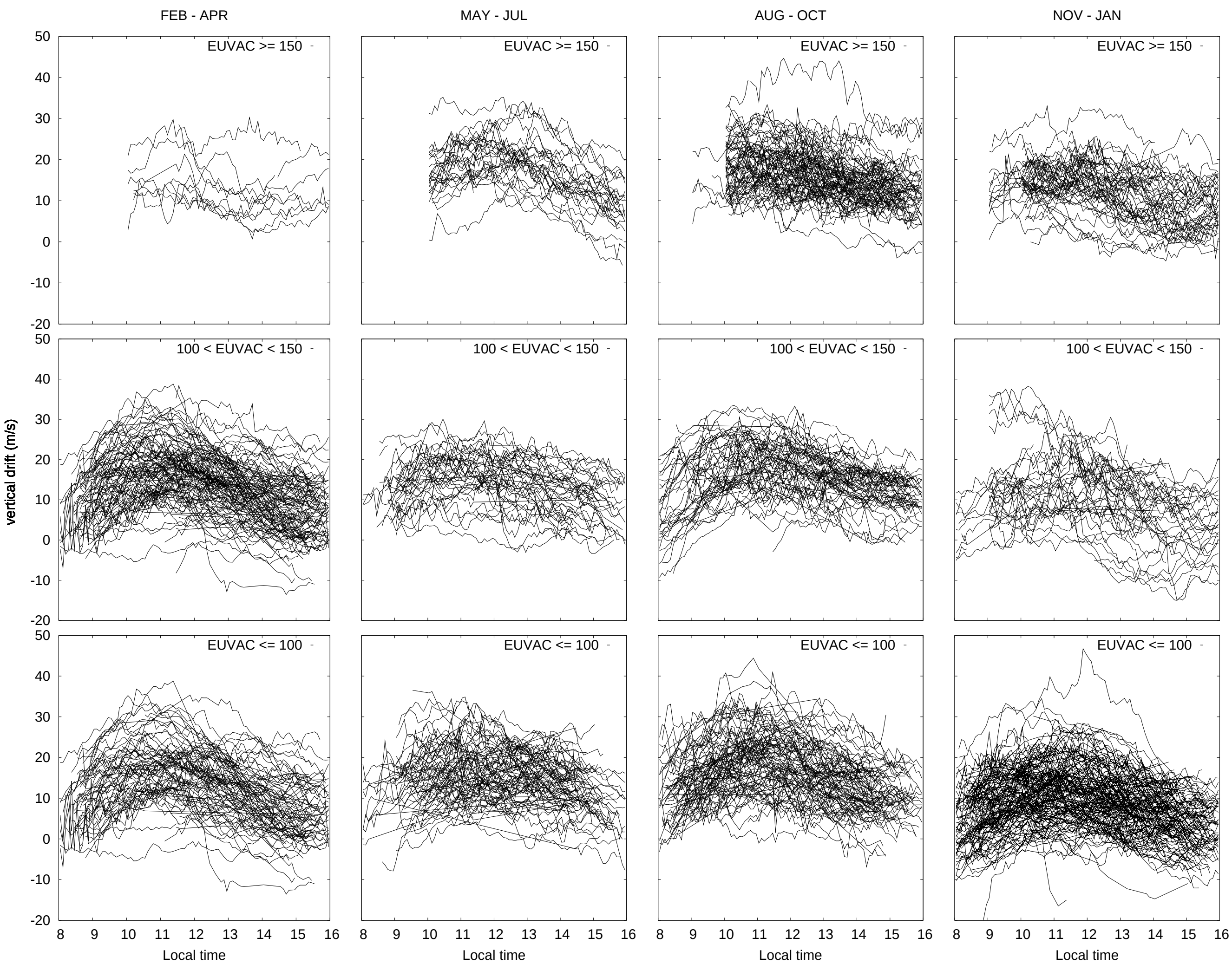

Figure 1. Local time profiles of JULIA quiet-time (Kp $\leq$ 3) vertical drifts for low, medium and high solar activity and different seasons.

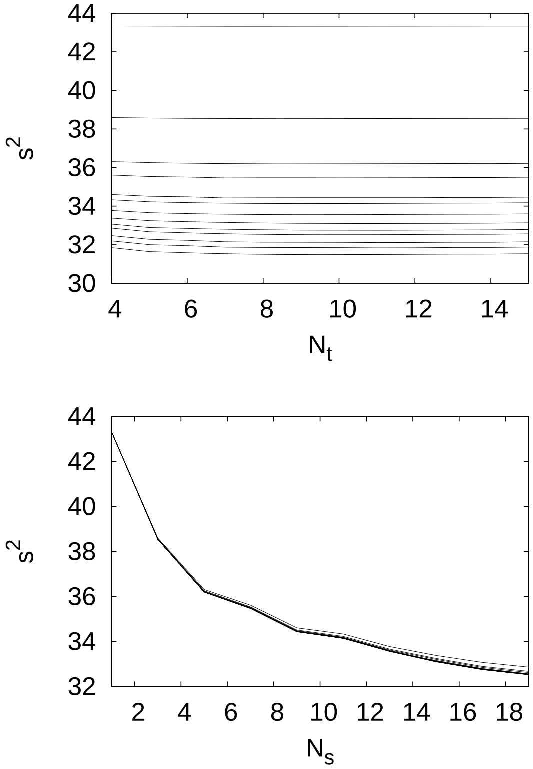

Figure 2. Residual mean square as a function of number of basis functions for each parameter of local time and season.

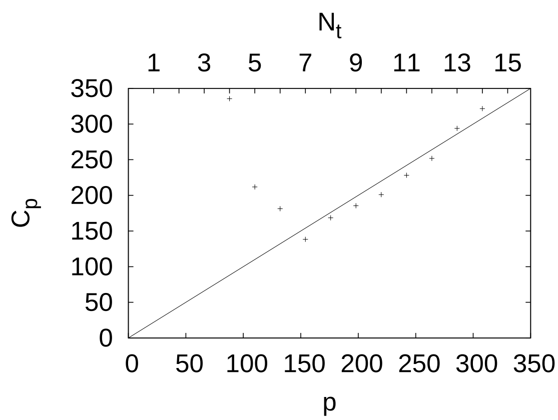

Figure 3. Mallows C_p statistic as a function of the total number of model parameters p, along with the line $C_p = p$.

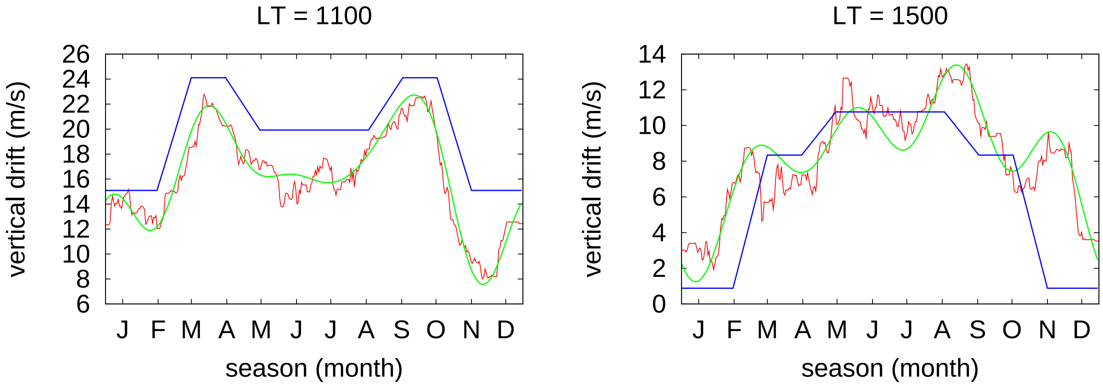

Figure 4. Comparison of Scherliess/Fejer model output (blue), JVDM model output (green) and raw JULIA data smoothed with a moving average of 30 days (red) as a function of season. An EUVAC index of 80 was used for the model outputs and the JULIA data was selected for EUVAC $<$ 100 and with a local time window of $\pm$ 1/2 hour around 1100 (left) and 1500 (right) local times.

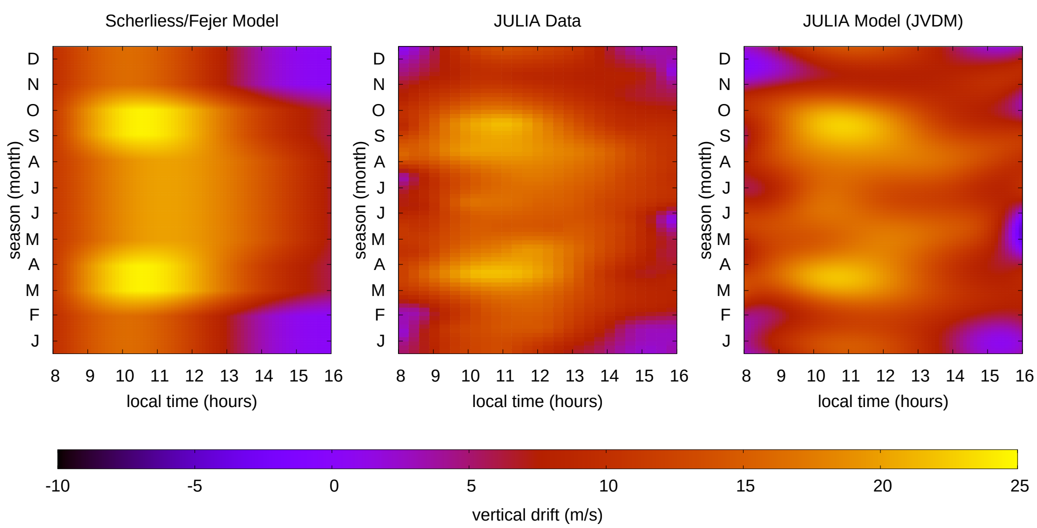

Figure 5. Scherliess and Fejer model output (left), raw JULIA vertical drift data (middle) and JULIA Vertical Drift Model output using EUVAC index of 80 (right).

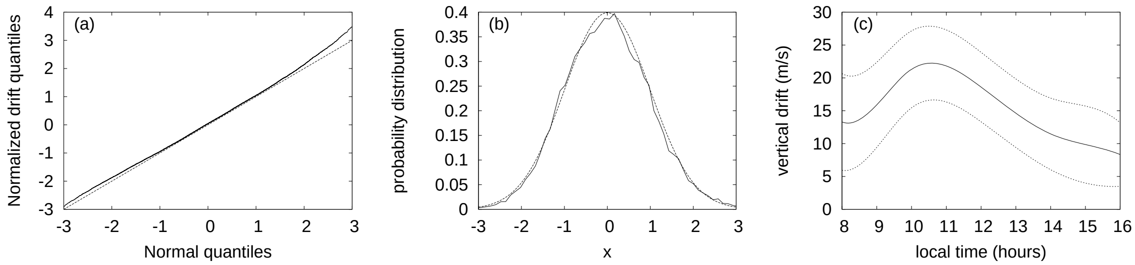

Figure 6. (a) Quantile-quantile plot with normalized drift quantiles vs normal quantiles. (b) Probability density function of normalized drift data along with ideal normal density profile. (c) Vertical drift local time profile (solid) with +/- standard deviation curves (dashed) for March equinox and EUVAC index of 100.